

Hong Kong Mathematics Olympiad (2006 – 2007)

Heat Event (Group)

香港數學競賽 (2006 – 2007)

初賽項目(團體)

除非特別聲明，答案須用數字表達，並化至最簡。

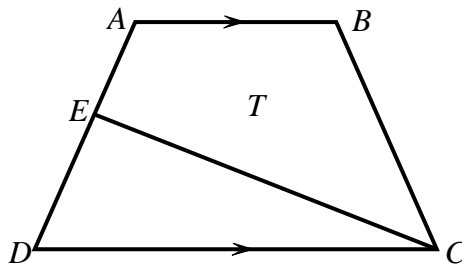
Unless otherwise stated, all answers should be expressed in numerals in their simplest form.

1. 若由 1 至 50 內與 50 互質的整數有  $N$  個，求  $N$  的值。(註：正整數  $a$  與  $b$  稱為互質若  $a$  與  $b$  的最大公因數是 1。)

If there are  $N$  integers from 1 to 50 that are relatively prime to 50, find the value of  $N$ . (Remark: positive integers  $a$  and  $b$  are said to be relatively prime if their greatest common divisor is 1.)

2. 如圖一，在梯形  $ABCD$  中， $AB \parallel CD$ ， $\angle BCE = \angle ECD$ ， $CE \perp AD$  及  $DE = 2AE$ 。若  $\triangle DEC$  的面積是  $2007 \text{ cm}^2$  及四邊形  $ABCE$  的面積是  $T \text{ cm}^2$ ，求  $T$  的值。

In Figure 1,  $ABCD$  is a trapezium,  $AB \parallel CD$ ,  $\angle BCE = \angle ECD$ ,  $CE \perp AD$  and  $DE = 2AE$ . If the area of  $\triangle DEC$  is  $2007 \text{ cm}^2$  and the area of quadrilateral  $ABCE$  is  $T \text{ cm}^2$ , find the value of  $T$ .



圖一

Figure 1

3. 已知  $a^2 - 3a + 1 = 0$ 。若  $A = \frac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$ ，求  $A$  的值。

Given that  $a^2 - 3a + 1 = 0$ . If  $A = \frac{2a^5 - 5a^4 + 2a^3 - 8a^2 + 7a}{3a^2 + 3}$ , find the value of  $A$ .

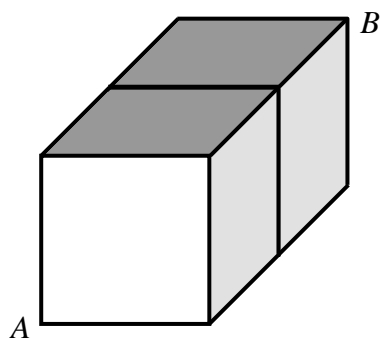
4. 已知點  $A$ 、 $B$  及  $C$  的坐標分別為  $(3, 4)$ 、 $(6, -4)$  及  $(8, 10)$ 。  $M$  及  $N$  分別為  $AB$  及  $BC$  的中點。  $X$  為  $AN$  上一點使得  $AX:XN=2:1$ 。 若  $r = \frac{CX}{XM}$ ，求  $r$  的值。

Given that the coordinates of the points  $A$ ,  $B$  and  $C$  are  $(3, 4)$ ,  $(6, -4)$  and  $(8, 10)$  respectively.  $M$  and  $N$  are the midpoints of  $AB$  and  $BC$  respectively.  $X$  is a point on  $AN$  such that  $AX:XN=2:1$ .

If  $r = \frac{CX}{XM}$ , find the value of  $r$ .

5. 如圖二，兩個邊長為  $1\text{ cm}$  的正方體組成一個  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$  的長方體。一隻螞蟻沿著長方體爬行，其爬行路線須為正方體的稜。牠從頂點  $A$  出發，以每分鐘爬行  $1\text{ cm}$  的速度，於 4 分鐘後到達頂點  $B$ 。若螞蟻可行路線數目共有  $S$  個，求  $S$  的值。

In Figure 2, a  $1\text{ cm} \times 1\text{ cm} \times 2\text{ cm}$  rectangular box is made up by two cubes with side length  $1\text{ cm}$ . An ant is climbing along the box in a way that it must stay on the edges of the cubes through out the climbing. Starting from vertex  $A$  and climbing with a speed of  $1\text{ cm}$  per minute, it reaches vertex  $B$  after 4 minutes. If the total number of possible paths taken by the ant is  $S$ , find the value of  $S$ .



圖二  
Figure 2

6. 若以 5 除  $7^{2007}$  所得的餘數是  $R$ ，求  $R$  的值。

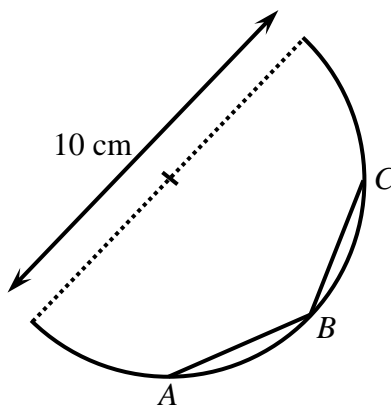
If the remainder of  $7^{2007}$  when dividing by 5 is  $R$ , find the value of  $R$ .

7. 設  $k = \sin 30^\circ + \cos 60^\circ + \sin 90^\circ + \cos 120^\circ + \cdots + \sin 1890^\circ + \cos 1920^\circ$ ，求  $k$  的值。

Let  $k = \sin 30^\circ + \cos 60^\circ + \sin 90^\circ + \cos 120^\circ + \cdots + \sin 1890^\circ + \cos 1920^\circ$ , find the value of  $k$ .

8. 如圖三，已知半圓的直徑為 10 cm。A、B 和 C 是半圓上任意的三點使 B 在弧 AC 上。設  $x$  為線段 AB 及 BC 的長度之和，求  $x$  可取的最大值。

In figure 3, given that the diameter of the semicircle is 10 cm. A, B and C are three arbitrary points on the semicircle where B is on the arc AC. If  $x$  is the sum of the length of the line segments AB and BC, find the greatest possible value of  $x$ .



圖三

Figure 3

9. 在坐標平面上，點  $A = (-6, 2)$ 、 $B = (-3, 3)$ 、 $C = (0, n)$  及  $D = (m, 0)$  組成一個四邊形 ABCD。求  $n$  的值使得該四邊形 ABCD 的周界為最短。

In the coordinate plane, the points  $A = (-6, 2)$ ,  $B = (-3, 3)$ ,  $C = (0, n)$  and  $D = (m, 0)$  form a quadrilateral ABCD. Find the value of  $n$  so that the perimeter of the quadrilateral ABCD is the least.

10. 已知整數  $x$  及  $y$  滿足  $3x + 5y = 1$ 。若  $S = x - y$  及  $S > 2007$ ，求  $S$  可取的最小值。

Given that integers  $x$  and  $y$  satisfying the equation  $3x + 5y = 1$ . If  $S = x - y$  and  $S > 2007$ , find the least possible value of  $S$ .